Name: _____

**Math 10550, Final Exam:**
**December 17, 2008**

Instructor: _____

- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.
- No calculators are to be used.
- The exam lasts for two hours.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 14 pages of the test.

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Name: _____

Instructor: _____

Multiple Choice

**1.**(6 pts.) Find the limit

$$\lim_{x\to 0} \frac{3-\sqrt{x+9}}{x}.$$

(a) $-\frac{1}{6}$

(b) $-3$

(c) The limit does not exist.

(d) $\frac{1}{6}$

(e) 3

**2.**(6 pts.) Find all points where the following function is discontinuous

$$f(x) = \begin{cases} \frac{(x-1)(x+2)}{(x^2-1)x} & x \neq 1 \\ \frac{3}{2} & x = 1 \end{cases}.$$

(a) $x = -2, x = -1, x = 1$

(b) $x = 0, x = -1$

(c) $x = 0, x = 1$

(d) $x = 0, x = -2, x = 1$

(e) $x = 0, x = -1, x = 1$

Name: _____

Instructor: _____

**3.**(6 pts.) If

$$f(x) = \sqrt{1 + \sqrt{1 + x}},$$

then $f'(8) =$

(a) $\frac{1}{8}$ (b) $\frac{1}{9}$ (c) $\frac{1}{24}$ (d) $\frac{1}{2}$ (e) $\frac{1}{12}$

**4.**(6 pts.) The second derivative of

$$f(x) = \frac{\sin x}{x}$$

is

(a) $\dfrac{-x^2 \sin x + 4x \cos x + 5 \sin x}{x^3}$

(b) $\dfrac{-x^2 \sin x - 3x \cos x + 2 \sin x}{x^3}$

(c) $\dfrac{x^2 \sin x + 4x \cos x + 2 \sin x}{x^3}$

(d) $\dfrac{-x^2 \sin x - 2x \cos x + 2 \sin x}{x^3}$

(e) $\dfrac{-x^2 \sin x - 3x \cos x + 3 \sin x}{x^3}$

Name: _____

Instructor: _____

**5.**(6 pts.) A body travels along a straight line according to the law

$$s = -t^4 - 4t^3 + 20t^2, \quad t \geq 0.$$

At what position, **after** the motion gets started, does the body first come to rest?

(a) $s = 36$ (b) $s = 24$ (c) $s = 2$

(d) $s = 32$ (e) $s = 12$

**6.**(6 pts.) Find an equation for the tangent line to

$$f(x) = \tan(x^2 + 2x)$$

at the point $(0, 0)$.

(a) $y = 2x$ (b) $y = 0$ (c) $y = \sqrt{2}x$

(d) $y = 2\sqrt{2}x$ (e) $y = -2x$

Name: _____

Instructor: _____

**7.**(6 pts.) Find an equation for the tangent line to the curve

$$x^3 + y^3 = 4xy$$

at the point $(2, 2)$.

(a) $y = 2x - 2$ (b) $y = x$ (c) $y = -x + 4$

(d) $y = -x - 4$ (e) $y = -2x + 6$

**8.**(6 pts.) The length of a rectangle is increasing at a rate of 8 cm/sec and its width is increasing at a rate of 3 cm/sec. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

(a) $140\,\text{cm}^2/\text{sec}$. (b) $211\,\text{cm}^2/\text{sec}$. (c) $190\,\text{cm}^2/\text{sec}$.

(d) $11\,\text{cm}^2/\text{sec}$. (e) $24\,\text{cm}^2/\text{sec}$.

Name: _____

Instructor: _____

**9.**(6 pts.) Use linear approximation to estimate

$$\frac{1}{\sqrt{3.9}}.$$

(a) $\frac{1}{\sqrt{3.9}} \approx \frac{9}{20}$ (b) $\frac{1}{\sqrt{3.9}} \approx \frac{1}{2}$ (c) $\frac{1}{\sqrt{3.9}} \approx \frac{81}{160}$

(d) $\frac{1}{\sqrt{3.9}} \approx \frac{11}{20}$ (e) $\frac{1}{\sqrt{3.9}} \approx \frac{79}{160}$

**10.**(6 pts.) Let

$$f(x) = x^3 + 3x^2 - 24x.$$

Find the absolute maximum and absolute minimum values of $f$ on the interval $[0, 10]$.

(a) Max at $x = 4$; Min at $x = 0$. (b) Max at $x = 10$; Min at $x = 0$.

(c) Max at $x = 4$; Min at $x = 2$. (d) Max at $x = 10$; Min at $x = 2$.

(e) Max at $x = 4$; Min at $x = 1$.

Name: _____

Instructor: _____

**11.**(6 pts.) Find the local and absolute maximum and minimum of

$$f(x) = 3x^{2/3} - x.$$

(a) Local min at $x = 1/8$; absolute min at $x = 1$; no absolute max.

(b) Local min at $x = 1$; local max at $x = 1/8$; no absolute min; absolute max at $x = -27$.

(c) Absolute min at $x = 0$; absolute max at $x = 8$.

(d) Local min at $x = 0$; local max at $x = 8$; no absolute max or min.

(e) Local max at $x = 1$; no absolute max; absolute min at $x = 0$.

**12.**(6 pts.) Let

$$f(x) = x^{5/3} - 5x^{2/3}.$$

On what intervals is $f$ concave up?

(a) $(-1, 0) \cup (0, \infty)$ (b) $(-8, 8)$ (c) $(1, \infty)$

(d) $(-\infty, -1)$ (e) $(0, 8)$

Name: _____

Instructor: _____

**13.**(6 pts.) Evaluate the limit

$$\lim_{x\to\infty}\left(\sqrt{x^2+2x}-x\right).$$

(a) $-\infty$ (b) $0$ (c) $1$ (d) $2$ (e) $\infty$

**14.**(6 pts.) The equation of the slant asymptote of the curve $y=\dfrac{2x^2+1}{x+1}$ is:

(a) $y=2x$ (b) $y=2x-2$ (c) $y=-2x+2$

(d) $y=x+2$ (e) $y=2x+2$

Name: _____

Instructor: _____

**15.**(6 pts.) Suppose the line $y = 4x - 2$ is tangent to the curve $y = f(x)$, when $x = 1$. If the Newton's method is used to locate a root of the equation $f(x) = 0$ and the initial approximation is $x_1 = 1$, find the second approximation $x_2$

(a) $-4$ (b) $1$ (c) $0$ (d) $2$ (e) $1/2$

**16.**(6 pts.) Calculate the following definite integral

$$\int_1^5 (5-x)^2 dx =$$

(a) $16$ (b) $-\frac{64}{3}$ (c) $3$ (d) $-16$ (e) $\frac{64}{3}$

Name: _____

Instructor: _____

**17.**(6 pts.) Let $g(x) = \int_{\sin x}^{0} t^2\,dt$. Find $g'(x)$.

(a) $-(\cos x)^2 \cos x$

(b) $-(\sin x)^2 \cos x$

(c) $(\cos x)^2 \cos x$

(d) $-(\sin x)^2 \sin x$

(e) $(\sin x)^2 \cos x$

**18.**(6 pts.) Calculate the integral $\int_0^2 \frac{x}{\sqrt{x^2+1}}\,dx$

(a) $\sqrt{5}-1$

(b) $-\sqrt{5}-1$

(c) $1-\sqrt{5}$

(d) $\sqrt{5}$

(e) $4$

Name: _____

Instructor: _____

**19.**(6 pts.) Which of the following is a Riemann sum corresponding to the integral

$$\int_0^1 \big(\tan x + 2\big)dx.$$

(a) $2 + \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} \tan(\frac{i}{n})$

(b) $\dfrac{2}{n} + \dfrac{2}{n}\displaystyle\sum_{i=1}^{n} \tan(\frac{i}{n})$

(c) $\dfrac{1}{n}\displaystyle\sum_{i=1}^{n} \left(\tan(\frac{i}{n}) + 2\right)$

(d) $\dfrac{2}{n}\displaystyle\sum_{i=1}^{n} \tan(\frac{2i}{n})$

(e) $\dfrac{1}{2n}\displaystyle\sum_{i=1}^{n} \tan(\frac{2i}{n})$

**20.**(6 pts.) The point on the line $6x + y = 9$ that is closest to the origin has $x$-coordinate

(a) $x = \dfrac{3}{2}$ (b) $x = 0$ (c) $x = 1$

(d) $x = \dfrac{44}{9}$ (e) $x = \dfrac{54}{37}$

Name: _____

Instructor: _____

**21.**(6 pts.) The curves $y = x^4 - 3$ and $y = -x^4 + 5$ enclose an area. Set up a definite integral which calculates the area of this region.

(a) $\int_{-1}^{1} (8 - 2x^4)\, dx$

(b) $\int_{0}^{\sqrt{3}} (8 - 2x^4)\, dx$

(c) $\int_{-1}^{1} 2\, dx$

(d) $\int_{-\sqrt{2}}^{\sqrt{2}} 2\, dx$

(e) $\int_{-\sqrt{2}}^{\sqrt{2}} (8 - 2x^4)\, dx$

**22.**(6 pts.) The plane region bounded below by the graph of $y = x$ and above by the graph $y = \sqrt{x}$ is rotated about the line $x = 5$. Which integral below gives the volume?

(a) $\pi \int_{0}^{1} (5 - \sqrt{x})^2 - (5 - x)^2 dx$

(b) $\pi \int_{0}^{1} (5 - x)^2 - (5 - \sqrt{x})^2 dx$

(c) $2\pi \int_{0}^{1} (x - 5) \cdot (\sqrt{x} - x) dx$

(d) $2\pi \int_{0}^{1} (5 - x) \cdot (x - \sqrt{x}) dx$

(e) $2\pi \int_{0}^{1} (5 - x) \cdot (\sqrt{x} - x) dx$

Name: _____

Instructor: _____

**23.**(6 pts.) Consider the plane region bounded by the graphs of $y = \sqrt{x}$, $y = 0$ and $x = 1$. Rotate this region about the line $y = -3$ and calculate the volume.

(a) $\frac{3\pi}{3}$ (b) $\frac{9\pi}{2}$ (c) $\frac{7\pi}{2}$ (d) $\frac{15\pi}{2}$ (e) $\frac{27\pi}{2}$

**24.**(6 pts.) Find the average of $f(x) = \sin^2(x) \cdot \cos(x)$ over $[0, \frac{\pi}{2}]$.

(a) $\frac{2}{3\pi}$ (b) $\frac{2}{\pi}$ (c) $\frac{1}{3}$

(d) $\frac{1}{\pi}$ (e) $\frac{1}{3\pi}$

Name: _____

Instructor: _____

**25.**(6 pts.) A (vertical) cylindrical tank has a height 1 meter and base radius 1 meter. It is filled full with a liquid with a density 100 kg/m$^3$. Find the work required to empty the tank by pumping all of the liquid to the top of the tank.

(a) 0 kg-m

(b) $200\pi$ kg-m

(c) $50\pi$ kg-m

(d) $500\pi$ kg-m

(e) $100\pi$ kg-m

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